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From this equation we get

$$AE^2 = \frac{AG^2 - 2AG\cdot AF\cos c + AF^2}{\sin^2 c}.$$

Substituting the values of AF and AG from (3),

$$AE^2 = \frac{m^2\cos^2 b - 2m^2\cos a\cos b\cos c + m^2\cos^2 a}{\sin^2 c}.$$

Then from (2),

$$\begin{aligned}
CE^2 &= \frac{m^2}{\sin^2 c}(\sin^2 c - \cos^2 b + 2\cos a\cos b\cos c - \cos^2 a)\\
&= \frac{m^2}{\sin^2 c}(\sin^2 c + \cos^2 c - \cos^2 b - \cos^2 c + \cos^2 b\cos^2 c - \cos^2 b\cos^2 c\\
&\qquad + 2\cos a\cos b\cos c - \cos^2 a)\\
&= \frac{m^2}{\sin^2 c}\{(1 - \cos^2 b - \cos^2 c + \cos^2 b\cos^2 c) - \cos^2 a + 2\cos a\cos b\cos c - \cos^2 b\cos^2 c\}\\
&= \frac{m^2}{\sin^2 c}\{(1 - \cos^2 b)(1 - \cos^2 c) - \cos^2 a + 2\cos a\cos b\cos c - \cos^2 b\cos^2 c\}\\
&= \frac{m^2}{\sin^2 c}\{\sin^2 b\sin^2 c - \cos^2 a + 2\cos a\cos b\cos c - \cos^2 b\cos^2 c\}\\
&= \frac{m^2}{\sin^2 c}\{\sin b\sin c + (\cos a - \cos b\cos c)\}\{\sin b\sin c - (\cos a - \cos b\cos c)\}\\
&= \frac{m^2}{\sin^2 c}\{\cos a - \cos(b + c)\}\{\cos(b - c) - \cos a\}\\
&= \frac{m^2}{\sin^2 c}\left\{2\sin\left(\frac{a+b+c}{2}\right)\sin\left(\frac{b+c-a}{2}\right)\right\}\left\{\sin\left(\frac{a-b+c}{2}\right)\sin\left(\frac{a+b-c}{2}\right)\right\}\\
&= \frac{4m^2}{\sin^2 c}\sin s\cdot\sin(s - a)\sin(s - b)\sin(s - c),
\end{aligned}$$

where

$$s = \frac{a+b+c}{2}.$$

Hence, $CE = \dfrac{2m}{\sin c}\sqrt{\sin s\sin(s - a)\sin(s - b)\sin(s - c)}$. Substituting this value of CE in (1), we have

$$v = 2lmn\sqrt{\sin s\sin(s - a)\sin(s - b)\sin(s - c)}.$$

Also solved by GEORGE W. HARTWELL, HORACE OLSON, J. A. CAPARO, and J. W. CLAWSON.

CALCULUS.

370. Proposed by PAUL CAPRON, Annapolis, Maryland.

The surface of a right circular cone having the semi-vertical angle α is cut by two planes, which intersect the axis at the same point, one at right angles to the axis, the other making the angle $(90^\circ - \beta)$ with the axis. Show that if the lateral surface of the right cone is S_1 and that of the oblique cone S_2,

$$S_2 = \sum_1^\infty T_n, \qquad \text{where} \qquad T_1 = S_1,\ T_{n+1} = T_n \times \frac{2n+1}{2n}(\tan\alpha\tan\beta)^2.$$

SOLUTION BY THE PROPOSER.

Let the vertex of the cone be the origin, its axis the z-axis, and let the intersection of the oblique plane with the xy-plane be parallel to the y-axis. Let the two planes cut the z-axis at $(0, 0, h)$, and let the radius of the right section be a. Then,

$$\sqrt{x^2 + y^2} = r, \text{ and } y/x = \tan\theta.$$

We have the following equations:

For the cone, $z = k \cot\alpha$, and for the oblique plane, $z = h - x\tan\beta x$. Eliminating z, we have for the projection on the xy-plane of the oblique section,

$$r = \frac{h\tan\alpha}{1 + \tan\alpha\tan\beta\cos\theta}.$$

If dA is an element of area in the xy-plane, the element of the conical surface of which it is the projection is $\csc\alpha \cdot dA$. Hence $dS_2 = \csc\alpha\, rd\theta dr$, and

$$S_2 = \tfrac{1}{2} h^2 \csc\alpha\tan^2\alpha \int_0^{2\pi} \frac{d\theta}{(1 + \tan\alpha\tan\beta\cos\theta)^2}$$

$$= \frac{h^2}{2}\sec\alpha\tan\alpha \int_0^{2\pi} (1 + k\cos\theta)^{-2} d\theta,$$

where

$$k = \tan\alpha\tan\beta.$$

$$\int_0^{2\pi} (1 + k\cos\theta)^{-2} d\theta = \int_0^{2\pi} (1 - 2k\cos\theta + 3k^2\cos^2\theta - \cdots) d\theta$$

$$= 2\pi\left[1 + \frac{3}{2}k^2 + \frac{15}{8}k^4 + \frac{35}{16}k^6 + \cdots\right],$$

where the general term is

$$T_{n+1} = (2n+1)k^{2n}\int_0^{2\pi} \cos^{2n}\theta d\theta = \frac{(2n+1)k^{2n}(2n-1)(2n-3)\cdots 1}{2n(2n-2)\cdots 2}\frac{\pi}{2} \times 4.$$

Hence,

$$T_{n+1} = T_n \times \frac{2n+1}{2n} k^2,$$

where

$$k = \tan\alpha\tan\beta.$$

$$T_1 = \pi k^2 \tan\alpha\sec\alpha = \pi a^2 \csc\alpha = S_1.$$

Also solved by Albert N. Nauer.

371. Proposed by B. F. FINKEL, Drury College.

Prove that the shortest distance between two curves or surfaces is normal to each.

Solution by Elijah Swift, University of Vermont.

Let us prove this for two surfaces; the proof for two curves is similar. Suppose the equations of the surfaces are $f(x, y, z) = 0$, $F(x, y, z) = 0$, and that the shortest distance is between the points (x_1, y_1, z_1) on f and (x_2, y_2, z_2) on F. Then we wish to make $(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2$ a minimum with the auxiliary conditions $f(x_1, y_1, z_1) = 0$, $F(x_2, y_2, z_2) = 0$. Necessary conditions are that the partial derivatives of